Eur. Phys. J. C 22, 269–276 (2001)
Digital Object Identifier (DOI) 10.1007/s100520100815

THE EUROPEAN PHYSICAL JOURNAL C

# Fragmentation functions from semi-inclusive DIS pion production and implications for the polarized parton densities

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Received: 6 August 2001 / Revised version: 3 October 2001 /
Published online: 12 November 2001 – 

**Abstract.** By combining recent HERMES data on semi-inclusive DIS (SIDIS) $\pi^\pm$-production with the singlet fragmentation function $D_\Sigma^{\pi^+}$, which is well determined from $e^+e^-$ data, we are able to extract, for the first time, the flavoured fragmentation functions $D_u^{\pi^+}$, $D_d^{\pi^+}$ and $D_s^{\pi^+}$ without making any assumptions about favoured and unfavoured transitions. Whereas $D_u^{\pi^+}$ and $D_d^{\pi^+}$ are very well determined, the accuracy of $D_s^{\pi^+}$ is limited by the uncertainty in evolving $D_\Sigma^{\pi^+}$ from the $Z^0$ pole down to the SIDIS scale of a few $(GeV)^2$. We discuss how the precision on $D_s^{\pi^+}$ could be improved. Knowledge of the $D_{q=u,d,s}^{\pi^+}$ will permit the extraction of the polarized parton densities from future polarized SIDIS asymmetry measurements. We study the precision that can be expected in such an extraction.

## 1 Introduction

Fully inclusive deep inelastic scattering (DIS) of the neutral type

$$l^\pm + N \to l^\pm + X \tag{1}$$

yields information only on the combination of parton densities $q(x)+\bar{q}(x)$, in the unpolarized case, and on the combination of polarized parton densities $\Delta q(x)+\Delta\bar{q}(x)$ when longitudinally polarized leptons interact with longitudinally polarized nucleons. It is crucially from reactions with neutrinos and antineutrinos, in the unpolarized case, that a separate knowledge of the parton $q(x)$ and antiparton $\bar{q}(x)$ densities can be inferred. This avenue is, at present, not open to the polarized case.

The main approach to a separate knowledge of the $\Delta q(x)$ and $\Delta\bar{q}(x)$ thus rests upon the growing activity in the field of polarized semi-inclusive deep-inelastic experiments of the type

$$\overrightarrow{l}^\pm + \overrightarrow{N} \to l^\pm + h + X \tag{2}$$

where $h$ is the detected hadron.

The cross sections (or spin asymmetries) for such reactions depend, in leading order QCD, upon products of parton densities and fragmentation functions (FFs) $D_q^h(z)$ for a parton $q$ to fragment into hadron h. (In NLO QCD these products become convolutions.)

It has been shown [1,2] that if systematic errors can be well enough controlled so as to allow a meaningful combination of data from different targets and hadrons of different charge, there is sufficient information to extract information on both the polarized parton densities and the fragmentation functions.

In the past this has not been possible and the strategy adopted in the analysis of the experimental data [3–5] has been to assume a complete knowledge of the unpolarized densities $q(x)$, $\bar{q}(x)$ and of the fragmentation functions $D_q^h(z)$, $D_{\bar{q}}^h(z)$. With these, in [4] an auxiliary function was constructed, the flavour ($q = u, d, s, \bar{u}, \bar{d}, \bar{s}$) purity $P_{q/N}^h(x, z)$ [6] for each hadron $h$ and for each target nucleon $N$. Given the purities, the polarized data can then, in principle, be used to directly extract the polarized densities $\Delta q(x)$, $\Delta\bar{q}(x)$.

However, the situation has now changed because recently the HERMES group has for the first time published unpolarized charge separated data for $\pi^\pm$ production on a proton target [7]. The main aim of our paper is to demonstrate that this data, taken in conjunction with the information on the flavour singlet combination $D_\Sigma^{\pi^+}$ of FFs which can be fairly reliably obtained from the data on $e^+e^- \to \pi^\pm X$ at the $Z^0$ peak, allows a first direct determination of the FFs $D_u^{\pi^+}$, $D_d^{\pi^+}$ and $D_s^{\pi^+}$. We note the caveat that the data in [7] covering $0.2 < z < 0.9$ exhibit large $\mathcal{O}(40\%)$ isospin violations at large $z \gtrsim 0.7$. It is most unlikely that such a large breaking of isospin

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invariance can be a genuine effect in the current fragmentation region of semi-inclusive DIS (SIDIS), and it would seem unnatural to incorporate it into an FF formalism. A possible explanation for the effect is given in [7]. Our analysis will therefore be relevant mainly for intermediate $0.2 < z \lesssim 0.7$.

It turns out that the least well determined FF is $D_s^{\pi^+}$, since it is most dependent on the evolution downwards of $D_\Sigma^{\pi^+}(z,Q^2)$ all the way from $Q^2 \approx m_{Z^0}^2$ to $Q^2 \approx$ few $(GeV)^2$, and this involves mixing with the gluon FF $D_G^{\pi^+}(z,Q^2)$. We try to assess the sort of accuracy required in future $e^+e^-$ measurements in order to achieve an accuracy of 20–30% on $D_s^{\pi^+}(z)$.

In order to study the accuracy of the polarized parton densities obtained in the past via the use of purity [4] we proceed as follows. Firstly, we use our FFs to calculate the central value and errors of the integrated purity function $\int dz\ P_{q/p}^{\pi^+}(x,z)$ and $\int dz\ P_{q/p}^{\pi^-}(x,z)$ for a proton target. Then, because at present the polarized SIDIS data does not exist separately for $\pi^+$ and $\pi^-$, we take the central values of the published polarized parton densities of Leader, Sidorov and Stamenov [8] obtained purely from DIS data, and using the central values of the purity we generate fake "data" on polarized SIDIS asymmetries $\Delta A_p^{\pi^+}$ and $\Delta A_p^{\pi^-}$, and also on the polarized DIS asymmetries. We then go through a similar procedure as adopted by the HERMES group to obtain from this "data" the polarized parton densities, with this difference, that we allow for the uncertainty in the value of the purity function. In this way we obtain an indication of the uncertainty in the polarized parton densities inherent in the purity approach.

In this paper we work to leading order (LO) in QCD as the purity concept only makes sense in LO and because in LO we can deal with simple algebraic equations which are physically most transparent to interpret and have a well-defined error propagation. Of course, in the long run, a more complete NLO analysis will be required. Standard experimental techniques [4] are based on an *ad hoc* combination of LO (polarized) parton distributions with e.g. LUND-type Monte Carlo fragmentation functions. Such an effective approach cannot be extended to NLO without a highly non-trivial definition of the long- and short distance pieces in the MC environment which is – to our knowledge – lacking at present. It will, therefore, be vital to bring the measurements in touch with well-defined factorized[1] QCD approaches [9] combining universal parton distribution functions (PDFs) with universal FFs because, otherwise, the extracted PDFs will not have any physical relevance. The most important experimental information will be on scheme- and model-independent data for cross sections and not on the extracted (unobservable) PDFs and FFs.

[1] The factorization of $x-$ and $z-$dependence at LO is an artefact from the one particle phase space (delta-function) of LO-SIDIS and is of no fundamental importance as opposed to the mass factorization of NLO cross sections

## 2 Extraction of fragmentation functions

### 2.1 Formalism

For a leading order treatment we follow the notation of [2] and remove some kinematical factors by introducing for the DIS and SIDIS cross sections on a proton target:

$$\tilde{\sigma}^{DIS} = \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^2\sigma^{DIS}}{dx\,dy} \tag{3}$$

$$\Delta\tilde{\sigma}^{DIS} = \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{y}{2-y}\right)\left[\frac{d^2\sigma^{DIS}_{++}}{dx\,dy}-\frac{d^2\sigma^{DIS}_{+-}}{dx\,dy}\right] \tag{4}$$

$$\tilde{\sigma}^{h} = \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^3\sigma^{h}}{dx\,dy\,dz} \tag{5}$$

$$\Delta\tilde{\sigma}^{h} = \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{y}{2-y}\right)\left[\frac{d^3\sigma^{h}_{++}}{dx\,dy\,dz}-\frac{d^3\sigma^{h}_{+-}}{dx\,dy\,dz}\right] \tag{6}$$

Here, $P^\mu$ and $l^\mu$ are the nucleon and lepton four momenta, and $\sigma_{\lambda\nu}$ refers to a lepton of helicity $\lambda$ and a nucleon of helicity $\nu$. The variables $x, y, z$ are the usual DIS kinematic variables. Then one has the very simple LO results:

$$\tilde{\sigma}^{DIS}(x,Q^2) = \sum_{q,\bar{q}} e_q^2\, q_i(x,Q^2) \tag{7}$$

$$\Delta\tilde{\sigma}^{DIS}(x,Q^2) = \sum_{q,\bar{q}} e_q^2\, \Delta q_i(x,Q^2) \tag{8}$$

$$\Delta\tilde{\sigma}^{h}(x,z,Q^2) = \sum_{q,\bar{q}} e_q^2\, \Delta q_i(x,Q^2)\, D_i^h(z,Q^2) \tag{9}$$

$$\tilde{\sigma}^{h}(x,z,Q^2) = \sum_{q,\bar{q}} e_q^2\, q_i(x,Q^2)\, D_i^h(z,Q^2), \tag{10}$$

Note that the inclusion of a factor $(1+R)/(1+\gamma^2)$ in (10) (see e.g. (5) of [4]) is not justified theoretically. The correct handling of the longitudinal cross-section is a more complicated NLO effect in SIDIS (see (56) - (60) of [2]). Here, as mentioned, we work to LO only. Specializing now to $\pi^\pm$ production we introduce the measured observables[2]

$$R_p^{\pi^\pm}(x,z,Q^2) \equiv \frac{\sigma_p^{\pi^\pm}}{\sigma_p^{DIS}} \tag{11}$$

$$= \frac{\tilde{\sigma}_p^{\pi^\pm}}{\tilde{\sigma}_p^{DIS}} \tag{12}$$

Using charge conjugation and isospin invariance we require only 3 independent FFs:

$$D_u^{\pi^+}(z,Q^2), \qquad D_d^{\pi^+}(z,Q^2), \qquad D_s^{\pi^+}(z,Q^2) \tag{13}$$

The remaining ones are then:

$$D_{\bar{u}}^{\pi^-} = D_d^{\pi^-} = D_{\bar{d}}^{\pi^+} = D_u^{\pi^+} \tag{14}$$

$$D_{\bar{d}}^{\pi^-} = D_u^{\pi^-} = D_{\bar{u}}^{\pi^+} = D_d^{\pi^+} \tag{15}$$

$$D_{\bar{s}}^{\pi^-} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_s^{\pi^+} \tag{16}$$

[2] As we are considering positive and negative charges seperately we are *not* adopting the convention $h^\pm \equiv h^+ + h^-$ [10–12]

Thus

$$R_p^{\pi^+} = \frac{1}{\tilde{\sigma}_p^{DIS}} \left\{ \frac{4}{9} \left( u D_u^{\pi^+} + \bar{u} D_{\bar{u}}^{\pi^+} \right) + \frac{1}{9} \left( d D_d^{\pi^+} + \bar{d} D_{\bar{d}}^{\pi^+} + s D_s^{\pi^+} + \bar{s} D_{\bar{s}}^{\pi^+} \right) \right\} = \frac{1}{9\tilde{\sigma}_p^{DIS}} \left\{ (4u + \bar{d}) D_u^{\pi^+} + (4\bar{u} + d) D_d^{\pi^+} + (s + \bar{s})\, D_s^{\pi^+} \right\} \quad (17)$$

Similarly

$$R_p^{\pi^-} = \frac{1}{9\tilde{\sigma}_p^{DIS}} \left\{ (4\bar{u} + d) D_u^{\pi^+} + (4u + \bar{d}) D_d^{\pi^+} + (s + \bar{s})\, D_s^{\pi^+} \right\} \quad (18)$$

Assuming a good knowledge of the unpolarized parton densities we can immediately obtain

$$D_u^{\pi^+} - D_d^{\pi^+} = \frac{9 \left( R_p^{\pi^+} - R_p^{\pi^-} \right) \tilde{\sigma}_p^{DIS}}{4u_V - d_V} \quad (19)$$

In order to obtain $D_u^{\pi^+} + D_d^{\pi^+}$ and $D_s^{\pi^+}$ we require one further piece of experimental information. We shall argue that it can be obtained from the data on $e^+e^- \to \pi^\pm X$ at the $Z^0$ peak.

## 2.2 Use of the $e^+e^-$ data

For some time it was believed that the fragmentation functions obtained by Binnewies et al. [10], from a detailed analysis of the $e^+e^-$ data over a wide range of energies, were reasonably well determined. However, recent analyses [11–13] have shown that equally good fits to $e^+e^-$ data can be achieved with FFs of a given flavour which differ widely from each other. The $e^+e^-$ data do not, therefore, constrain the FFs of a given flavour very well, and, in retrospect, this is really not surprising.

However, by a piece of good fortune, the $e^+e^-$ data at the $Z^0$ peak directly measure a linear combination of FFs which is very close to the $\mathrm{SU(3)_f}$ flavour singlet combination, i.e. in

$$D_{\mathrm{meas}}^{\pi^+ + \pi^-} = \sum_{q=u,d,s} \left( D_q^{\pi^+ + \pi^-} + D_{\bar{q}}^{\pi^+ + \pi^-} \right) \hat{e}_q^2(s) \quad (20)$$

the squared electroweak couplings $\hat{e}_q^2(s)$ from $\mathrm{SU(2)} \times \mathrm{U(1)}$ gauge symmetry (given e.g. in the appendix of [11]) are flavour-independent to within $\sim 25\%$ at $\sqrt{s} = M_Z$ as opposed to a relative factor of 4 for the electromagnetic couplings of up- and down-type quarks at lower cms energies. The exact singlet below in (22) would correspond to a measurement at an $e^+e^-$ cms energy of $\sqrt{s} = 78.4$ GeV or $\sqrt{s} = 113.1$ GeV where it happens that $\hat{e}_u^2(s) = \hat{e}_d^2(s) = \hat{e}_s^2(s)$. Accordingly (20) is approximately proportional to the singlet combination

$$D_\Sigma^{\pi^+} \equiv \left( D_u^{\pi^+} + D_{\bar{u}}^{\pi^+} + D_d^{\pi^+} + D_{\bar{d}}^{\pi^+} + D_s^{\pi^+} + D_{\bar{s}}^{\pi^+} \right) \quad (21)$$

$$= 2 \left( D_u^{\pi^+} + D_d^{\pi^+} + D_s^{\pi^+} \right) \quad (22)$$

where we have used charge conjugation and eqs.(14) - (16) in the last step. Using isospin and charge conjugation invariance $D_u^{\pi^\pm} + D_{\bar{u}}^{\pi^\pm} = D_d^{\pi^\pm} + D_{\bar{d}}^{\pi^\pm}$ and approximating $\hat{e}_u^2(s)/\hat{e}_d^2(s)\big|_{s=M_Z^2} = \hat{e}_u^2(s)/\hat{e}_s^2(s)\big|_{s=M_Z^2} \simeq 3/4$ we can write the singlet combination

$$D_\Sigma^{\pi^+} = \frac{4}{7} \tilde{D}_{\mathrm{meas}}^{\pi^+ + \pi^-} - \frac{1}{7} \left( D_s^{\pi^+} + D_{\bar{s}}^{\pi^+} \right) \quad (23)$$

where we have introduced a convenient change in normalization

$$\tilde{D}_{\mathrm{meas}}^{\pi^+ + \pi^-} = D_{\mathrm{meas}}^{\pi^+ + \pi^-} \;/\; \hat{e}_d^2(s) \quad (24)$$

The *extreme* limits $0 < (D_s^{\pi^+} + D_{\bar{s}}^{\pi^+}) < (D_u^{\pi^+} + D_{\bar{u}}^{\pi^+})$ then correspond to

$$\frac{4}{7}\, \tilde{D}_{\mathrm{meas}}^{\pi^+ + \pi^-} < D_\Sigma^{\pi^+} < \frac{6}{11}\, \tilde{D}_{\mathrm{meas}}^{\pi^+ + \pi^-} \quad (25)$$

i.e. to only a $\sim 5\%$ uncertainty for $D_\Sigma^{\pi^+}$.

Not surprisingly, therefore, the singlet FFs in the analyses [11–13] agree with each other to better than 5% for $0.2 < z < 0.7$ as seen in Fig. 1. We may thus take as a known quantity $D_\Sigma^{\pi^+}(z, Q^2 = m_{Z^0}^2)$ and from Fig. 1 we observe a stable evolution down to $Q^2 = 100$ GeV$^2$.

But we require this quantity at a scale of a few (GeV)$^2$ and it thus has to be evolved down through a large range of $Q^2$, and in this evolution mixes with the poorly known gluon FF $D_G^{\pi^+}$. (Of course we cannot carry out the evolution of $D_{\mathrm{meas}}$ itself since it is a combination of singlet and non-singlet pieces and we do not know the values of these separately.) The FF analyses in [10–13] cover data down to $\sqrt{s} \simeq 30$ GeV and from Fig. 1 we judge this fixes a stable singlet FF down to $\sqrt{s} \simeq 10$ GeV. Below, however, the evolution uncertainties set in and from the right of Fig. 1 we quantify this uncertainty conservatively to be a $\sim 20$ % effect uniformly in $z$. We convinced ourselves this is indeed a typical order of magnitude by comparing the several sets of LO and NLO FFs for $\pi^+ + \pi^-, K^+ + K^-, h^+ + h^-$ in [10–13] and not only the two sets plotted in Fig. 1. Clearly, a low scale measurement of the singlet FF or a resolution of the evolution ambiguities through a determination of the gluon FF would be highly desirable information.

Subject therefore to possible errors due to the evolution, we have available the additional experimental data that we require, and we then obtain:

$$D_u^{\pi^+} + D_d^{\pi^+} = \frac{9 \left( R_p^{\pi^+} + R_p^{\pi^-} \right) \tilde{\sigma}_p^{DIS} - 2s\, D_\Sigma^{\pi^+}}{4(u + \bar{u} - s) + d + \bar{d}} \quad (26)$$

and

$$D_s^{\pi^+} = \frac{-18 \left( R_p^{\pi^+} + R_p^{\pi^+ -} \right) \tilde{\sigma}_p^{DIS} + [4(u + \bar{u}) + d + \bar{d}]\, D_\Sigma^{\pi^+}}{2\,[4(u + \bar{u} - s) + d + \bar{d}\,]} \quad (27)$$

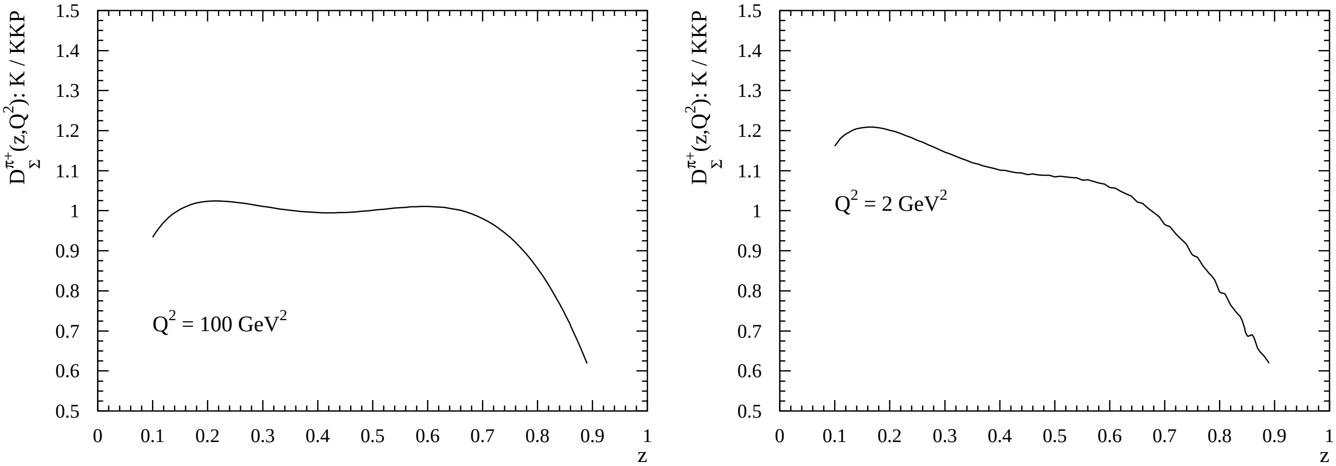


**Fig. 1.** The ratio of singlet fragmentation functions $D_\Sigma^{\pi^+}$ found in [11] (K) and [12] (KKP), at $Q^2 = 100$ GeV$^2$ (left) and at a typical SIDIS value $Q^2 = 2$ GeV$^2$ (right)

We note that the singlet FF plays no role in (19) and that its weight increases in going from (26), where it is multiplied by the suppressed strange PDF, to (27), where it is multiplied by the unsuppressed $u$ and $d$ PDFs. Correspondingly, we must expect an increasing importance of the propagation of the $\sim$ 20% error of $D_\Sigma^{\pi^+}$ into these equations.

The LH sides of (19),(26) and (27) are functions of $z$ and $Q^2$, whereas the RH sides are, in principle, functions of $x, z$ and $Q^2$. Only in the LO approximation does the variable $x$ become a *passive* variable [2] i.e. there is no dependence on it. Strictly one should test for this lack of $x$-dependence, as a measure of the reliability of the LO treatment. However, in this paper, in order to improve statistics, we shall take it for granted that the LO treatment is adequate.

### 2.3 Combined analysis of HERMES and $e^+e^-$ data

The formalism given in (14)-(16), (19), (22), (26) and (27) presupposes the availability of data at fixed $x$ and $y$. In fact the available HERMES data is integrated over the kinematic range [4] $Q^2 > 1$ (GeV/c)$^2$, $W^2 > 10$ GeV$^2$, $y < 0.85$. Handling integrated data slightly complicates the formalism since the $y$-dependent factors in the numerator and denominator of (11) no longer cancel out to give the simpler result (12). We have done the analysis using the data integrated over the kinematic range of the experiment and have checked that using the simpler formalism with [4]

$$x = \langle x \rangle = 0.082 \tag{28}$$

$$Q^2 = \langle Q^2 \rangle = 2.5\,(\mathrm{GeV/c})^2 \tag{29}$$

$$W^2 = \langle W^2 \rangle = 28.6\,(\mathrm{GeV})^2 \tag{30}$$

makes no discernible difference to the results for the FFs.

The stability of our results for the central values of $D_u^{\pi^+}$, $D_d^{\pi^+}$ is studied in Fig. 2. The NLO determination of

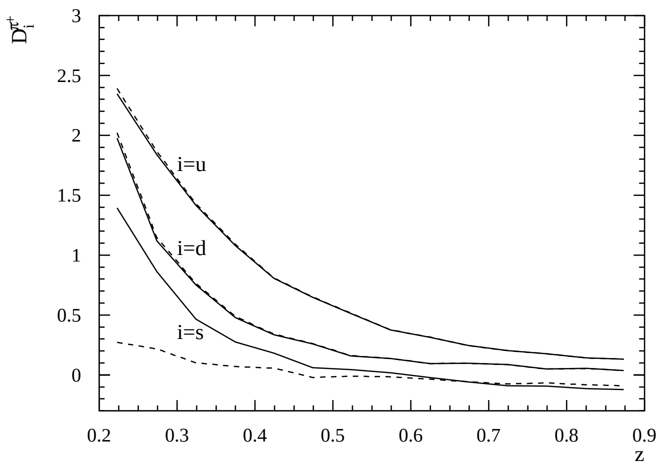


**Fig. 2.** The valence-type ($D_u^{\pi^+}$), sea-type ($D_d^{\pi^+}$) and strange-type ($D_s^{\pi^+}$) FF into charged pions extracted under the assumption of isospin-invariance from HERMES SIDIS measurements supplemented by the singlet FF of [11] (solid) and [12] (dashed), respectively. Details are given in the text. Switching between the LO $\leftrightarrow$ NLO parametrizations of [11,12] leads to similar variations in the FFs whereas the variation from employing different unpolarized PDFs is negligible

the singlet combination $D_\Sigma^{\pi^+}$ due to Kretzer [11] was utilized. To test the stability of our results we have used two different sets of unpolarized parton densities. We found the effect of employing, respectively, the NLO MRST [14] or the NLO GRV [15] unpolarized parton densities in (17), (18), (19), (26) and (27) leads to a negligible $\sim$ 5% effect. This is because the unpolarized densities are very well constrained in the region of interest. We have checked that use of the LO GRV densities also have almost no noticeable effect.

The FFs $D_u^{\pi^+}$ and $D_d^{\pi^+}$ are quite well constrained by the SIDIS data, but, as expected, $D_s^{\pi^+}$ is sensitive to the singlet combination of FFs determined from $e^+e^-$ data. This can be clearly seen in Fig. 2 where we compare the results for each $D_q^{\pi^+}$ using NLO versions of $D_\Sigma^{\pi^+}$ as obtained by Kretzer [11] and Kniehl, Kramer and Pötter [12] from the $e^+e^-$ data. As mentioned in the Introduction, $D_\Sigma^{\pi^+}$ is very well determined at the $Z^0$ peak, but the mixing, under evolution, with the gluon FF $D_G^{\pi^+}$ induces an uncertainty of about $10-20\%$ at $Q^2 = 2.5\ (\mathrm{GeV/c})^2$. As can be seen in Fig. 2, $D_s^{\pi^+}$ may even turn unphysically negative at large $z \gtrsim 0.7$ where our analysis is not supposed to be reliable, anyway, as mentioned in the Introduction. We note Fig. 2 shows the typical effect expected from the uncertainty of $D_\Sigma^{\pi^+}$ and that a similar picture emerges if we switch $D_\Sigma^{\pi^+}$ between the LO and NLO parametrizations of [11]: The stability of $D_u^{\pi^+}$ and $D_d^{\pi^+}$ is remarkable. $D_s^{\pi^+}$, on the other hand, changes significantly. If a measurement could fix $D_\Sigma^{\pi^+}(Q^2 \simeq 2.5\ \mathrm{GeV}^2)$ to within $\sim 5\%$ we would have a handle on $D_s^{\pi^+}$ as well at the $\sim 20-30\%$ level.

In summary we see that $D_u^{\pi^+}$ and $D_d^{\pi^+}$ are remarkably well constrained by the SIDIS data. $D_s^{\pi^+}$ however, is undetermined within a factor of about 2. In Fig. 3 we show the final results for our FFs and our estimate of their errors. These include Gaussian error propagation of the experimental errors in [4] combined with a 20% error of $D_\Sigma^{\pi^+}(Q^2 = 2.5\ \mathrm{GeV}^2)$. The FFs can be described analytically[3] at $\langle Q^2 \rangle = 2.5\mathrm{GeV/c}^2$ by

$$D_u^{\pi^+} = 0.689\ z^{-1.039}\ (1-z)^{1.241} \tag{31}$$

$$D_d^{\pi^+} = 0.217\ z^{-1.805}\ (1-z)^{2.037} \tag{32}$$

$$D_s^{\pi^+} = 0.164\ z^{-1.927}\ (1-z)^{2.886} \tag{33}$$

where $D_s^{\pi^+}$ has to be taken with care as Fig. 3 shows that basically no value within $0 < D_s^{\pi^+} < D_u^{\pi^+}$ can be excluded at present.

Finally, as a matter of interest, we compare in Fig. 4 the $D_{q=u,d,s}^{\pi^+}$ obtained in this paper with the LO $D_q^{\pi^+}$ obtained by Kretzer [11] purely from an analysis of the $e^+e^-$ data. In the latter the flavour separation is not fixed by the data and is somewhat *ad hoc* and assumed $D_d^{\pi^+} = D_s^{\pi^+}$ and $D_u^{\pi^+} > D_d^{\pi^+}$ by imposing $(1-z)\ D_u^{\pi^+} = D_d^{\pi^+}$ at the input scale. Surprisingly, Kretzer's $D_q^{\pi^+}$ are not very different from the $D_q^{\pi^+}$ obtained from our analysis of the SIDIS data!

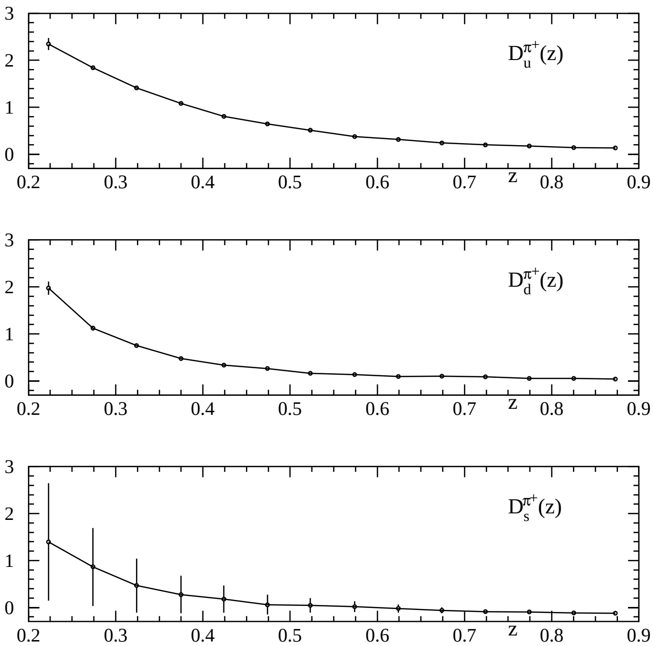


**Fig. 3.** The extracted fragmentation functions with errors which combine the experimental errors from [4] with a typical 20% uncertainty arising from the evolution of the singlet fragmentation function

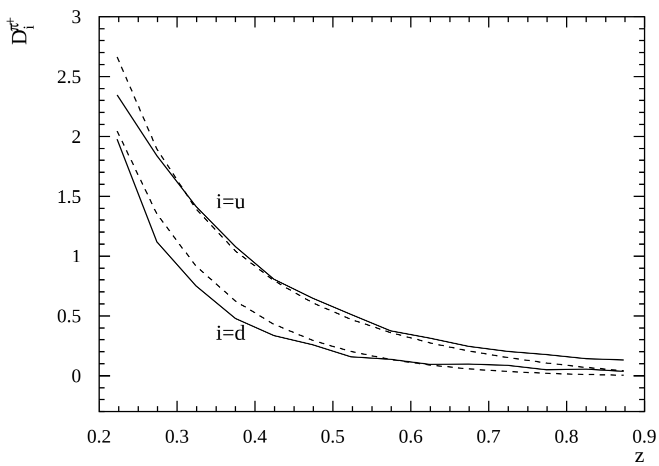


**Fig. 4.** A comparison of the extracted FFs $D_u^{\pi^+}$, $D_d^{\pi^+}$ (solid lines) at $\langle Q^2 \rangle = 2.5\ \mathrm{GeV}^2$ as in Fig. 1 to the LO-parametrization in [11] (dashed) where $(1-z)D_u^{\pi^+}(z) = D_d^{\pi^+}(z)$ at the input $Q_0^2$

## 3 Implications for the polarized parton densities

As mentioned in the Introduction, the absence of neutrino data for polarized DIS means that the extraction of the individual $\Delta q(x)$ is impossible. Only the combination $\Delta q(x) + \Delta\bar{q}(x)$ can be found and the flavour separation of these relies heavily on the evolution in $Q^2$ and is thus unreliable, given the small range of $Q^2$ available in polarized DIS experiments. Thus polarized SIDIS has a vital role to play in this matter.

At present, however, there are no published data for polarized $\pi^\pm$-production, though there does exist data for undifferentiated polarized $h^\pm$- production, which have been used by the HERMES group to extract information on the polarized parton densities via what is known as the purity method.

[3] Employing $D_q^{\pi^+} = N\ z^\alpha\ (1-z)^{\beta_q}$ ansätze with flavour independent $N, \alpha$ at $\langle Q^2 \rangle = 2.5\mathrm{GeV/c}^2$ slightly worsens the quality of the parametrization

We believe that in this approach the errors on the polarized parton densities are somewhat underestimated and we shall use our FFs to study this question.

The flavour $q$ purity function for protons [6] used by the HERMES group [4][4] is defined by

$$P^h_{q/p}(x) = \frac{e_q^2\, q(x)\, \int_{0.2}^1 D_q^h(z)\, dz}{\sum_{q'} e_{q'}^2\, q'(x)\, \int_{0.2}^1 D_{q'}^h(z)\, dz} \tag{34}$$

where again, we utilize the MRST parton densities and take $Q^2 = \langle Q^2 \rangle$.

Defining now the SIDIS spin asymmetry

$$\langle \Delta A_p^h(x) \rangle \equiv \frac{\int_{0.2}^1 dz\, \Delta\tilde{\sigma}_p^h(x,z)}{\int_{0.2}^1 dz\, \tilde{\sigma}_p^h(x,z)} \tag{35}$$

we have in LO,

$$\langle \Delta A_p^h(x) \rangle = \sum_q P^h_{q/p}(x) \left( \frac{\Delta q(x)}{q(x)} \right). \tag{36}$$

Similarly for the DIS spin asymmetry we can define

$$P^{DIS}_{q/p}(x) = \frac{e_q^2\, q(x)}{\sum_{q'} e_{q'}^2\, q'(x)} \tag{37}$$

and then, in LO, we have, with $Q^2 = \langle Q^2 \rangle$:

$$\frac{\Delta\tilde{\sigma}_p^{DIS}(x)}{\tilde{\sigma}_p^{DIS}(x)} = \sum_q P^{DIS}_{q/p}(x) \left( \frac{\Delta q(x)}{q(x)} \right). \tag{38}$$

Similar expressions for $\langle \Delta A_n^h(x) \rangle$ and $\Delta\tilde{\sigma}_n^{DIS}(x)/\tilde{\sigma}_n^{DIS}(x)$ can be obtained in an obvious way.

At each value of $x$ there are in principle 6 pieces of data ($h = \pi^\pm$ for $p$, $h = \pi^\pm$ for $n$, and DIS for $p$, $n$), so that there is enough information to *solve* for the 6 quark polarized densities $\Delta q(x)/q(x)$, for $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$. In the published analyses of the latter data [4] the HERMES group has preferred to model the polarized sea with assumptions such as

$$\frac{\Delta\bar{u}}{\bar{u}} = \frac{\Delta\bar{d}}{\bar{d}} = \frac{\Delta s}{s} = \frac{\Delta\bar{s}}{\bar{s}} \equiv \frac{\Delta q_s}{q_s} \tag{39}$$

or

$$\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s} \equiv \Delta q_s \tag{40}$$

and then to obtain the 3 independent polarized densities by making a best fit to the 6 -pieces of data at each $x$.

The problem with this approach is that the purity functions were constructed using LUND model information on the FFs. We think [2] this is an un-reliable procedure since a combination of (polarized) PDFs and LUND-type of FFs is at present lacking a rigorous theoretical framework as opposed to our combination of universal (polarized) PDFs with universal FFs in line with the factorization theorems of QCD [17].

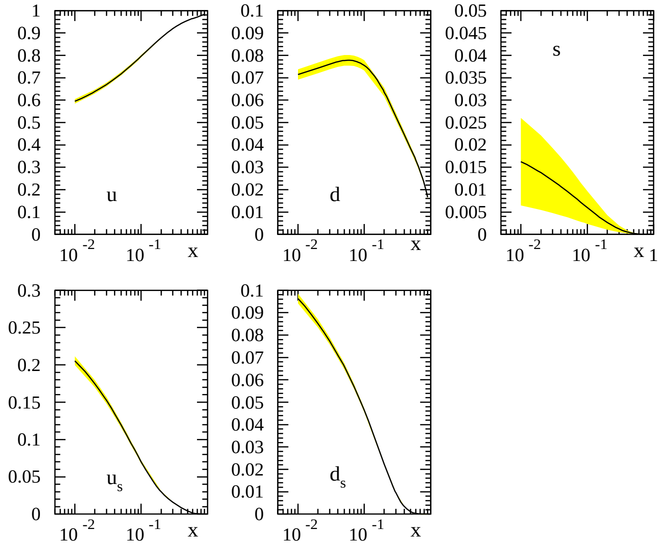


**Fig. 5.** Purity functions $P^{\pi^+}_{q/p}(x)$ extracted from HERMES data [7] with 20% uncertainty from the evolution of the flavour-singlet $D^{\pi}_{\Sigma}$

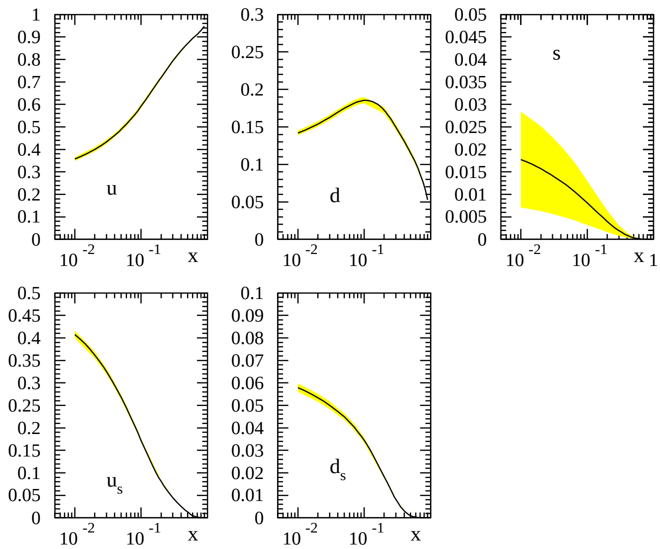


**Fig. 6.** Purity functions $P^{\pi^-}_{q/p}(x)$ extracted from HERMES data [7] with 20% uncertainty from the evolution of the flavour-singlet $D^{\pi}_{\Sigma}$

We argue that the above approach much underestimates the uncertainty on the polarized parton densities. To illustrate this we construct purity functions and their errors for pion production, using the fragmentation functions determined by us and the unpolarized MRST parton densities. The formulae are exactly as in (34), (35) and (36) with $h$ replaced by $\pi^+$. We show our calculated purities for protons with errors in Figs. 5, 6. The size of the errors are as to be anticipated from the errors on the $D^{\pi^+}_{q=u,d,s}$ in Fig. 3 and the definition of the purities in (34).

[4] Note that the graphs shown in the HERMES publications [16] and labelled "purity" are actually plots of an "effective purity" incorporating various experimental cuts

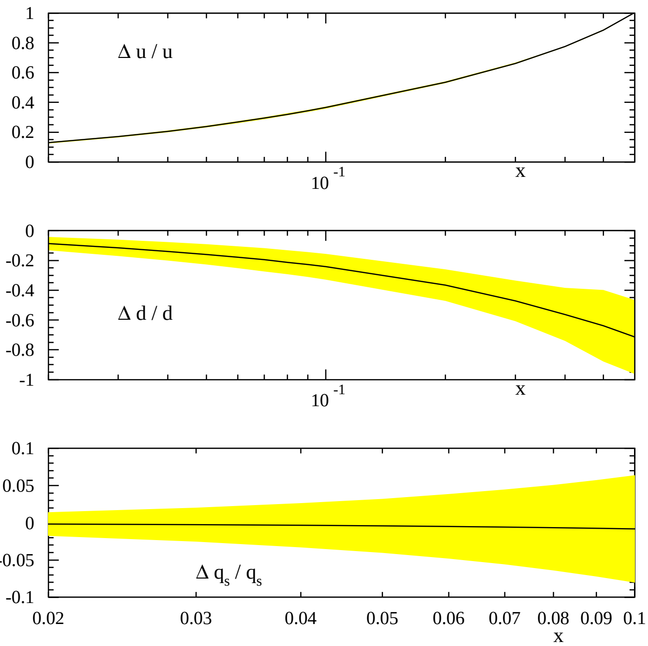


**Fig. 7.** Polarized parton densities extracted from the perfect "data" as explained in the text. The HERMES simplifying assumption $\Delta\bar{u}/\bar{u} = \Delta\bar{d}/\bar{d} = \Delta s/s = \Delta q_s/q_s$ has been used. The uncertainties arise solely from the realistic experimental errors on our fragmentation functions

In the absence of separate $\pi^\pm$ SIDIS spin asymmetry data, we take the central values of the polarized parton densities, as derived by Leader, Sidorov and Stamenov [8] from purely DIS data and by feeding these $\Delta q$ into (36) and (38) and into the analogous one for $\Delta A_p^{\pi^-}$, in which we utilize the central values of the purity functions, we generate a set of fake "data" for $\Delta A_p^{\pi^+}$, $\Delta A_p^{\pi^-}$ and $\Delta\tilde{\sigma}_p^{DIS}/\tilde{\sigma}_p^{DIS}$.

Having now this set of fake "data" we forget where it came from and use it to solve for the polarized parton densities, mimicking the approach used by the HERMES group. Thus we take

$$\frac{\Delta\bar{u}}{\bar{u}} = \frac{\Delta\bar{d}}{\bar{d}} = \frac{\Delta\bar{s}}{\bar{s}} \equiv \frac{\Delta q_s}{q_s} \qquad (41)$$

and solve (38), (35) (for $h = \pi^+,\ \pi^-$) for $\Delta u/u$, $\Delta d/d$ and $\Delta q_s/q_s$. In this analysis we treat the "data" as perfectly known, but include realistic errors on the purities, arising from the errors on our FFs. In this way we illustrate the uncertainty on the polarized parton densities arising solely from the uncertainties on the purity functions.

The results are shown in Fig. 7. It is seen that whereas $\Delta u/u$ is largely insensitive to the uncertainty on the purity, both $\Delta d/d$ and $\Delta q_s/q_s$ inherit significant errors from this uncertainty.

Bearing in mind that the errors shown in Fig. 7 arise solely from the uncetainty on the purities, one learns from this study that it is misleading to treat the purities as absolutely known quantities. It would be far more meaningful to follow the strategy suggested in [2] and use the SIDIS data to obtain both the FFs and the polarized parton densities. The purity is an unnecessary element and in any case loses its usefulness in NLO.

## 4 Conclusions

We have shown that a judicious combination of the HERMES SIDIS data on $\pi^\pm$ production and certain aspects of the data on $e^+e^- \to \pi^\pm X$ allows the extraction, for the first time, of the flavour separated fragmentation functions $D_u^{\pi^+}$, $D_d^{\pi^+}$ and $D_s^{\pi^+}$.

The key element in this approach is the avoidance of *any ad hoc* model-dependent flavour separation in the $e^+e^-$ data, by noting that at the $Z^0$ peak what is well determined is essentially the light flavour singlet combination of fragmentation functions $D_\Sigma^{\pi^+} = 2(D_u^{\pi^+} + D_d^{\pi^+} + D_s^{\pi^+})$, which is almost identical in all analyses of the $e^+e^-$ data. The negative aspect of this approach is the need to evolve $D_\Sigma^{\pi^+}$ down from the $Z^0$ region to the SIDIS region of a few $(GeV)^2$, which involves mixing with the poorly known gluon fragmentation function.

In fact the HERMES data is extremely accurate, so that almost all the uncertainty in our determination of $D_u^{\pi^+}$, $D_d^{\pi^+}$ and $D_s^{\pi^+}$ arises from the uncertainty in the evolution of $D_\Sigma^{\pi^+}$. As it turns out, this has little effect on $D_u^{\pi^+}$ and $D_d^{\pi^+}$, which are very well determined, but $D_s^{\pi^+}$ has relatively large errors.

We have also examined the question of the precision with which the polarized parton densities could be extracted from future polarized SIDIS pion production data. Here we have assumed perfect 'data', then followed the HERMES purity method to obtain the polarized parton densities, and thereby displayed the uncertainties generated solely by the errors on the purity functions. The significance of this study is that in the earlier analyses [3, 4] the purity functions are taken as almost perfectly known with essentially no errors. As expected we have found that $\Delta d$ and $\Delta q_s$ are significantly affected by the uncertainties in the purity functions. This suggests that in the published polarized parton densities extracted from polarized SIDIS $h^\pm$-production data [3–5], the uncertainties given are missing an inherent error arising from the fragmentation uncertainties as quantified in this paper.

*Acknowledgements.* We are grateful to Mark Beckmann and Helmut Böttcher for many stimulating discussions. This research was supported by a UK Royal Society Collaborative Grant and by the National Science Foundation under Grant PHY-0070443 and by Grant Ph-1010 of the Bulgarian National Science Foundation.

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